DETERMINATION OF AIRCRAFT TRANSFER FUNCTION ZEROES FROM IN-FLIGHT TRANSIENT RESPONSE CURVES

by

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SUMMARY

The purpose of this study was the examination of an empirical method of obtaining aircraft transfer functions directly from in-flight records without recourse to lengthy frequency domain techniques.

The proposed method utilizes a precision analog computer and plotter to generate response curves matching the in-flight records.

The computer set-up used for simulation is so arranged that each numerator coefficient of the transfer function is directly related to a potentiometer setting. At the most, two potentiometer settings are unknown, since the first and last coefficients can be obtained from the initial slope and steady state value by using Laplace initial and final value theory. In the case of longitudinal motion the period of the phugoid is of such a long duration as to make it impractical to obtain final value steady state conditions in actual flight records. This is not prohibitive for this problem solution method, however, since one coefficient can be determined from the flight record initial slope or suitable accelerometer measurement and then the two remaining coefficients are determined by adjusting potentiometer settings.

Utilization of this method indicated that errors would be

of the order of five per cent in magnitude with the combination of computer and recorder used. It is felt that with a high speed repetitive computer and a large high-persistence cathode-ray oscilloscope, the time of solution and degree of error might be reduced if sufficiently accurate flight data were obtained initially.

Further study is indicated for inputs differing from step functions such as input with the Laplace transform:

$$\delta_{(S)} = \frac{\delta}{S(S\tau + 1)}$$

The theory for this case is mentioned briefly in the Proposed Technique section.

CONTENTS

INTRODUCTION

Considerable theoretical work has been done with frequency domain techniques as applied to aircraft dynamic analysis, whereas little effort is indicated in the field of transfer function determination directly from in-flight transient response data. The purpose of this thesis is to investigate a simplified method of determining transfer functions by the latter means.

The empirical transfer function solutions are checked against the solutions of the equations of motion utilizing stability derivatives and other data of the F-86D aircraft. This aircraft was chosen since it is of relatively recent vintage and has been in existence long enough to supply reliable information sufficient to verify the techniques developed in this thesis. Transfer function poles are readily determined from flight test records; however, the zeroes require tedious computation when frequency domain methods are used. The values of the transfer function poles were taken from analytical computations, however, they were checked on the computer for validity and accuracy. The previous precaution was taken in order to narrow the field of investigation and source of error in determining the zeroes which were the crux of the overall problem. It was subsequently determined that this precaution was not necessary and would not be required in an actual practical situation.

Two basic theorems of the Laplace transformation (See Ref. 1) constitute an important part of the development:

(a) Final value theorem - If the function f(t) and its first derivative are Laplace transformable and if

$$L\left[f(t)\right] = F(s)$$

then

$$\lim_{s \to 0} sF(s) = \lim_{t \to \infty} f(t)$$

(b) Initial value theorem - If the function f(t) and its first derivative are Laplace transformable and if

$$L\left[f(t)\right] = F(s)$$

then

$$\lim_{s \to \infty} sF(s) \qquad \lim_{t \to 0} f(t)$$

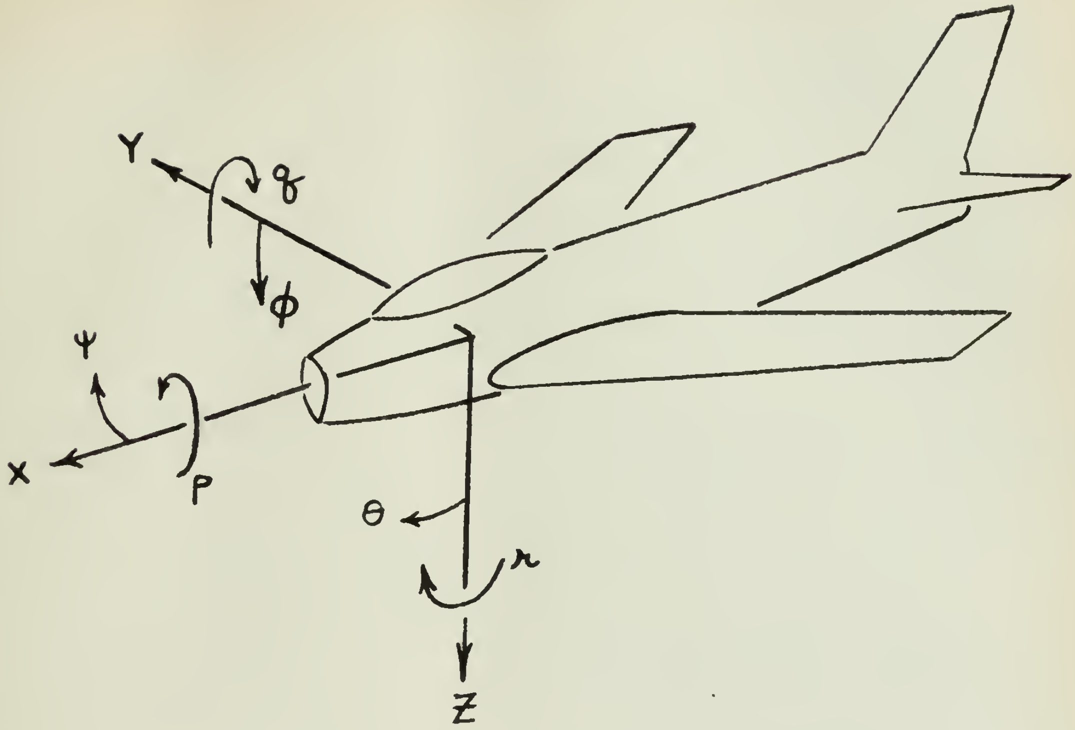
The application of these theorems will be shown in conjunction with measurements taken from the assumed in-flight transient response curves.

This study was undertaken by R. W. Illgen and W. P. Vossceler under the guidance and direction of Professor R. M. Howe as a joint thesis project to satisfy, in part, the requirements for a Master of Science Degree in Aeronautical Engineering at the University of Michigan, Ann Arbor, Michigan, during the period September 1957 - June 1958.

SYMBOLS AND NOTATION

α	Angle of attack, degrees or radians
b	Wing span, feet or inches
c	Chord length, feet or inches
C_D	Drag coefficient, D/qS
C_L	Lift coefficient, L/qS
C_{mac}	Moment coefficient about aerodynamic center
C_N	Normal force coefficient, N/qS
C_{N_α}	Normal force coefficient curve slope due to angle of attack $dC_N/d\alpha$
C_M	Pitch moment coefficient, m/qSd
$dC_L/d\alpha$	Slope of the lift curve
δ_e	Elevator deflection, degrees or radians
g	Acceleration of gravity, ft/sec^2
h	Altitude, feet
L	Lift, pounds
m	Mass of airplane, slugs
M	Mach number
q	Dynamic pressure, $\rho V^2/2$
S	Wing area, square feet
V	Airspeed, velocity, knots, mph, ft/sec
W	Weight of aircraft, pounds
β	Angle of attack in yaw plane, degrees or radians
ω_n	Natural frequency, rad/sec
ζ	Damping coefficient
τ	Time constant, sec

AIRCRAFT AXIS SYSTEM



Axis	X	Y	Z
Force	F_x	F_y	F_z
Moment	L'	M	N
Linear Velocity	u	v	w
Linear Acceleration	$\dot{u}$	$\dot{v}$	$\dot{w}$
Angular Displacement	ϕ	θ	ψ
Angular Velocity	p	q	r
Angular Acceleration	$\dot{p}$	$\dot{q}$	$\dot{r}$
Moment of Inertia	I_{xx}	I_{yy}	I_{zz}

$\bar{V}_p = \bar{i}U + \bar{j}V + \bar{k}W$ space linear velocity vector

$\bar{\Omega} = \bar{i}P + \bar{j}Q + \bar{k}R$ space angular velocity vector

ILLUSTRATIONS

PROPOSED TECHNIQUE

THEORY

Transfer functions can be obtained from the equations of motion which are ratios of polynomials of the general form:

$$\frac{X(s)}{\delta(s)} = \frac{b_n s^n + b_{n-1} s^{n-1} + \ldots\ldots\ldots b_o}{a_m s^m + a_{m-1} s^{m-1} + \ldots\ldots\ldots a_o} \qquad m > n$$

Multiplying the above by s^{m-n} and employing the Laplace initial value theorem for a step input:

$$\lim_{s \to \infty} s(s^{m-n})X = \lim_{t \to o} \frac{d^{m-n}X(t)}{dt} = \frac{b_n}{a_m} \delta$$

The slope of the (m-n-1) derivative response curve is then equal to $b_n \delta / a_m$.

The final value theorem for a step input gives:

$$\lim_{s \to 0} sX = \lim_{t \to \infty} X(t) = \frac{b_o}{a_o} \delta$$

The first and last coefficients of the numerator can now be easily evaluated since the denominator coefficients are found by the simple means of measuring damping and frequency. For $n \geq 2$ there are (n-1) unknown coefficients remaining.

If the above transfer is mechanized (n-1) potentiometers representing the unknown coefficients can be varied so as to obtain results matching the in-flight records. This is not overly difficult to accomplish since there are a maximum of two potentiometer settings which are unknown and these are known

at least to an order of magnitude. This would apply to rigid-body aircraft.

A suggested method of procedure for a step function input is to match phase fairly closely and then match initial slopes for the same steady state value. Iteration brings relatively rapid convergence.

For the case where the input transform $\delta(s)$ is $\dfrac{\delta}{s(s\tau\ 1)}$ the final value theorem gives results identical to the previous case. The initial value theorem, however, indicates that

$$\lim_{t \to 0} \frac{d^{m-n-1} X(t)}{dt^{m-n-1}} = \frac{b_n \delta}{\tau a_m}$$

where τ is the time constant of the input.

Only examples having step function inputs will be considered in this thesis.

EQUIPMENT

The computer used for these studies was a 20 amplifier electronic differential analyzer designed and built by the Department of Aeronautical Engineering, University of Michigan. A photograph of the computer, along with the Variplotter, is shown in Fig. 1. This computer had resistor accuracy of .02%, which was more than required for the present study. The computing amplifiers had a gain of approximately 10^5 and were manually balanced. Coefficient potentiometers were set to an accuracy of 0.1%.

The Pace Associates Model 1100D Variplotter used to plot the analogue computer outputs is a table-top size, self-balancing, potentiometer type of recorder. It records two variables simultaneously in the form of a rectangular coordinate graph. The static accuracy of both the pen and arm is .075 per cent. The dynamic accuracy for both is .2% for plotting speeds up to 15 inches per second.

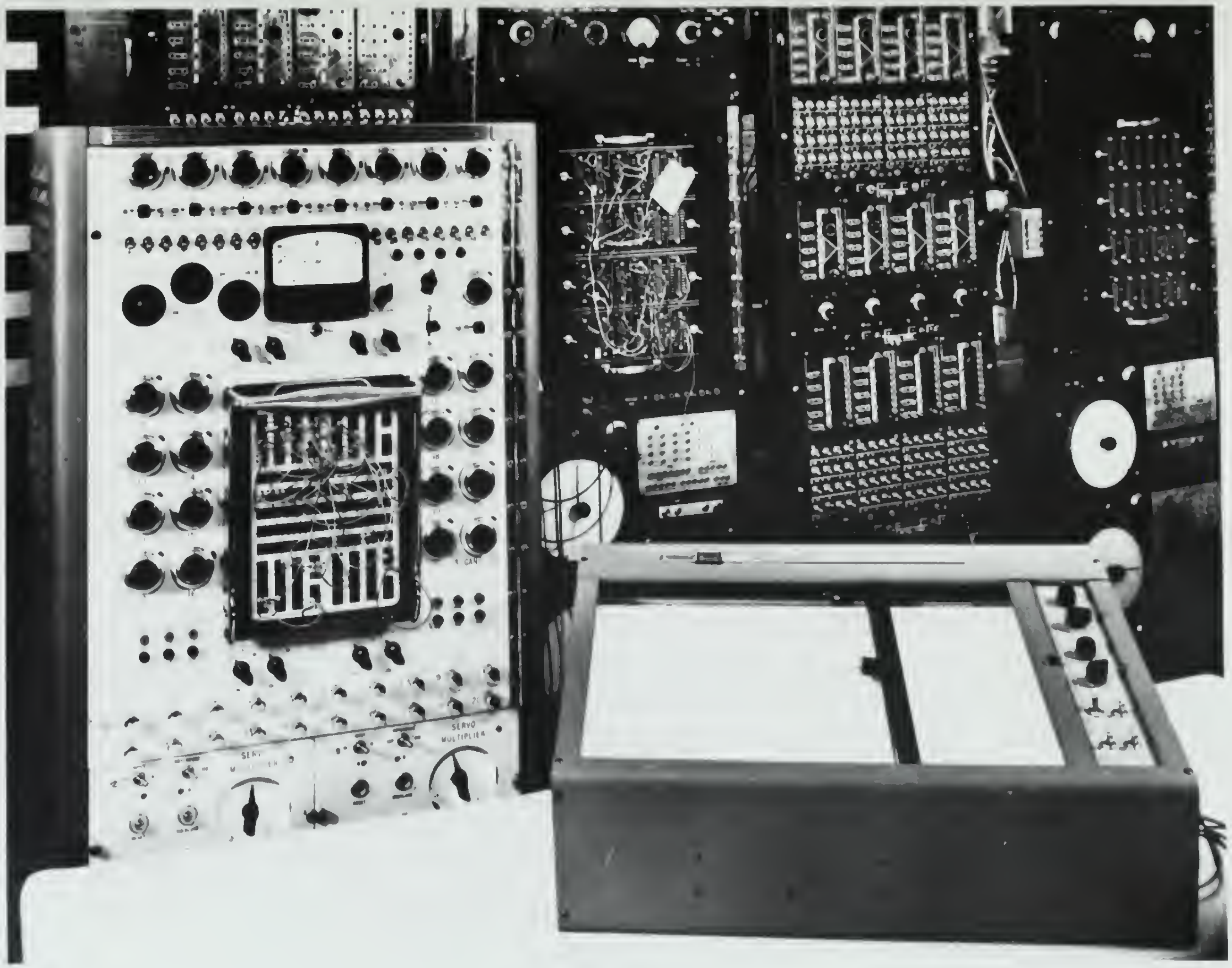


FIGURE 1

Computer and Plotter

EXAMPLES

Examples of the determination of zeroes for both longitudinal and lateral functions will be given.

I. Longitudinal Transfer Function Zeroes

Since no suitable flight records were available, flight data was assumed from solving the aircraft equations on an electronic differential analyzer.

Under conditions of perturbations from steady level flight at Mach .8 and 30,000 feet the following equations were utilized based on the airplane data in Appendix A.

$$\dot{u} = -.00664u + .0497w - 31.9_q - 32.2\,\theta$$

$$\dot{w} = -.0407u - 1.018w + 790.3_q - 1.29\,\theta - 1.55\,\delta_e$$

$$\dot{q} = .000475u - .0119w - .737_q \qquad - .413\,\delta_e$$

$$\dot{\theta} = q$$

Scaling the set of equations for use on the analog computer gives

$$\dot{u} = -.00664u + .00497(10w) - .319(100q) - .322(100\,\theta)$$

$$\dot{w} = -.0407u - .1018(10w) + 7.903(100q) - .0129(100\theta) -.155(10\delta_e)$$

$$100\,\dot{q} = .0476u - .119(10w) - .737(100q) -.0413(10\delta_e)$$

$$100\,\dot{\theta} = 100q$$

These equations were then mechanized as shown in Fig. 2 (see Appendix B for all remaining figures). The following scaling was used:

1 fps = .1 volts

1 rad = .1 volts

1 deg $=$ .1 volts

1 rad/sec $=$ .1 volts

The exact transfer function of $Y_{\theta\delta_e}$ was found from this set of equations to be:

$$Y = \theta/\delta_e = -\frac{23.6(s^2 + .983\,s + .00855)}{(s^4 + 1.762s^3 + 10.16s^2 + .0195s + .0311)}$$

Since there is very little coupling between the short period and the phugoid motion it was considered possible that the poles and zeroes of the above transfer would be the same as the poles and zeroes of the uncoupled transfer functions.

The approximate formula for the short period transfer function as based on Ref. 2 is

$$q/\delta_e = \frac{K_{q\delta_e}(1.032\,s + 1)}{(.0978\,s^2 + .1885\,s + 1)}$$

The approximate formula for the phugoid transfer function as obtained from the same source is

$$\theta/\delta_e = \frac{K_{\theta\delta_e}(127\,s + 1)}{(305.5\,s^2 + 2.405\,s + 1)}$$

The product of the above two expressions does indeed give a very close approximation to $Y_{\theta\delta_e}$ which makes a lengthy Newton Raphson approach (see Ref. 3) or other factorization technique unnecessary.

The parameters of the approximate transfer function are then as follows:

ω_{ns} =	3.22 radians	ω_{np} =	.0572 radians
T_s =	1.862 seconds	T_p =	109.5 seconds
ζ_s =	.304	ζ_p =	.0685

These values were used to assure that the response curves obtained from the mechanization of these curves were indeed correct. The computer response curves of Fig. 3 had the following parameters

ω_{ns} =	3.22 radians	ω_{np} =	.0572 radians
T_s =	2 seconds	T_p =	112 seconds
ζ_s =	.3	ζ_p =	.07

The above paramcters give the poles immediately in the canonical form.

The determination of zeroes for the transfer function $Y_{\theta \delta_e}$ was considered next.

In the interest of a simpler circuit, the constant term (.0311) in the denominator was assumed to be negligible. An 's' was then factored out of the remaining terms giving the transfer function of $Y_{q \delta e}$

$$q/\delta_e = \frac{-23.6(s^2 + .983s + .00855)}{(s^3 + 1.762s^2 + 10.16s + .0195)}$$

which has the dimensions of degree per degree or radian per radian.

The denominator is now a cubic with coefficients which can be equated to potentiometer settings and amplifier gains as follows for the longitudinal simulator computer circuit of Fig. 4:

$\alpha_1 K_1 K_2 = 1.762, \qquad K_1 = 1, \; K_2 = 10, \; {}_1 = .1762$

$\alpha_2 K_1 K_2 K_3 = 10.16, \qquad K_3 = 2, \; \alpha_2 = .508$

$\alpha_3 K_1 K_2 K_3 K_7 = .0915, \qquad (\alpha K)_7 = .1, \; \alpha_3 = .010$

Inasmuch as the steady state of the phugoid is difficult to obtain in flight tests, it was decided that a more practical method of procedure would be to determine the potentiometer setting affecting the initial slope by utilizing flight records and to vary the remaining two potentiometers.

The initial slope potentiometer setting corresponding to the coefficient of the s^2 term in the numerator was found in the following manner.

The initial slope is found from Fig. 3a or 3b. Recalling that

$$\dot{q}_o/\delta = b_2/a_3$$

we have from Fig. 3

$$b_2 = .4 \text{ rad/sec}^2\text{/degree.}$$

This was used to obtain the potentiometer setting as shown in Appendix C.

As has been indicated before, steady state for the phugoid takes too long to obtain and a cut and try approach is more expedient. This leaves potentiometers five and six which must be adjusted in the computer setup of Fig. 4. Various combinations over the full range of each potentiometer were tried. At each setting an output trace of q for a 1^o elevator step was compared

with the assumed flight record. Noting the effects and trends of these combined settings it was possible to come within 5% of what the actual required settings should be after only about 18 trials. Increased magnification using specialized high-persistence cathode ray oscilloscopes should increase aecuracy of sueh measurements.

Fig. 5 illustrates the technique in a general way utilizing the following sequenee of settings:

Trial	Potentiometer	
	5	6
A	.5	.5
B	.75	.5
C	1.0	.5
D	1.0	.0

It is to be noted that the initial slope remains unaffected as might be expected. The latter condition suggests that the potentiometer 4 setting could also have been obtained by trial and error matching instead of from initial slope measurement. Variation of potentiometer 5 affects the time and magnitude of the first peak as well as setting the continued level of the entire eurve. Variation of potentiometer 6 has a negligible effect on the initial peak, however, it raises or lowers the tail of the curve thus influencing the final or steady state value.

II. Lateral Transfer Function Zeroes

Again, since no suitable flight records were available, the flight response curves were obtained by solving the aircraft equations on an electronic differential analyzer.

In a manner similar to the longitudinal case, the airplane data in Appendix A was used to obtain

$$\dot{v} = -.179v + 31.8p - 796r + 32.2\phi$$

$$\dot{p} = -.435v - 3.6p + 1.295r + .313\dot{r} - 1.015\delta_a + .164\delta_r$$

$$\dot{r} = .0121v - .0389p - .388r + .0765\dot{p} - .0146\delta_a - .0838\delta_r$$

$$\dot{\phi} = p + .04r$$

Scaling the set of equations for use on the analog computer gives:

$$\dot{v} = -.179v + .318(100p) - 7.96(100r) + .322(100\phi)$$

$$100\dot{p} = -.0435v - 3.6(100p) + 1.295(100r) + .313(100\dot{r}) - 101.5\delta_a + 16.4\delta_r$$

$$100\dot{r} = 1.21v - .0389(100p) - .388(100r) + .0765(100\dot{p}) - 1.46\delta_a - 8.38\delta_r$$

$$100\dot{\phi} = 100p + .04(100r)$$

These equations were then mechanized as shown in Fig. 6. The scaling was as follows:

1 fps = .1 volt

1 rad = .1 volt

1 deg = .1 volt

A static check was performed by observing the voltage on inverters 1, 6, and 11 for various initial condition voltages. The response curves are shown in Fig. 7.

The important transfer functions for this set of equations were found to be as follows:

$$Y_{\phi \delta_r} = \frac{-.138(s^2 - 2.45s - 9.86)}{D(s)}$$

$$Y_{\phi \delta_a} = \frac{1.051(s^2 + .582s + 10.1)}{D(s)}$$

$$Y_{v\delta_r} = \frac{-62.7(s^2 + 4.06s - .0296)}{D(s)}$$

$$Y_{v\delta_a} = \frac{-42(s^2 - .872s - .325)}{D(s)}$$

$$Y_{r\delta_r} = \frac{.073(s^3 + 4.49s^2 + 1.45s + .752)}{D(s)}$$

$$Y_{r\delta_a} = \frac{.0945(s^3 + .321s^2 + 4.48s + 4.51)}{D(s)}$$

where

$$D(s) = s^4 + 4.16s^3 + 10.6s^2 + 38.5s - .01975.$$

Since the transfer functions for uncoupled rolling motion, spiral motion, and the dutch roll motion promised to be useful in finding the poles and zeroes of the above transfer functions these modes of motion were investigated. For uncoupled rolling motion we assumed that $r = \beta = \delta_r = 0$. This gives

$$\dot{p} = -3.6p - 1.015\delta_a$$

or

$$\phi/\delta_a = - \frac{.382}{s(.287s + 1)} .$$

For the dutch roll motion we assumed that $p = \phi = \delta_a = 0.$

This gives

$$\dot{v} = -.179v - 796r$$

$$\dot{r} = .0121v - .388r - .0838\,\delta_r$$

or

$$r/\delta_r = - \frac{.0838\,(s + .179)}{(s^2 + .567s + 9.72)}$$

For spiral motion we assumed that $\dot{v} = \dot{p} = \dot{r} = \dot{\beta} = \delta_a = \delta_r = 0.$ This gives

$$0 = -.179v + 31.8p - 796r + 32.2\,\phi$$

$$0 = -4.35v - 360p + 129.5r$$

$$0 = 1.21v - 3.89p - 38.8r$$

$$0 = \dot{\phi} - p - .04r$$

If we assume in addition that $v = p = 0$, we get a spiral mode time constant of 1875 seconds.

With these as approximations, synthetic division coupled with the Newton-Raphson iteration technique for determining roots was used to give

$$Y_{\phi\delta_r} = \frac{-.138(s - 3.267)(s + 3.022)}{D(s)}$$

$$Y_{\phi\delta_a} = \frac{1.051(s^2 + .582s + 10.1)}{D(s)}$$

$$Y_{v\delta_r} = \frac{-62.7(s + 4.06)\, s}{D(s)}$$

$$Y_{v\delta_a} = \frac{-42(s - 1.15)(s + .282)}{D(s)}$$

$$Y_{r\delta_r} = \frac{.073(s + 4.1)(s^2 + .32s + .2)}{D(s)}$$

$$Y_{r\delta_a} = \frac{.0945(s + .91)(s^2 - .579s + 5)}{D(s)}$$

where

$$D(s) = (s - .000513)(s + 3.95)(s + .2255s + 9.742)$$

The parameters associated with the denominator of the various uncoupled transfer functions are then as follows:

T_r = .287 seconds - the time constant of the uncoupled rolling motion.

T_s = 1875 seconds - the time constant of the spiral mode

$\omega_{n_{dr}}$ = .324 radians per second - dutch roll

ζ_{dr} = .875

The parameters associated with the denominator of the transfer function are found to be:

T_r = .253 seconds

T_s = 1950 seconds

$\omega_{n_{dr}}$ = .324

ζ_{dr} = .875

These parameters give the poles immediately in the

canonical form.

The parameters of the simulator circuit shown in Fig. 8 are determined by setting the coefficients equal to the potentiometer settings and amplifier gains as follows:

$\alpha_1 K_1 K_2 = 4.16$	$K_1 = 1,\ K_2 = 10$	$\alpha_1 = .416$
$\alpha_2 K_1 K_2 K_3 = 10.6$	$K_3 = 2,$	$\alpha_2 = .53$
$\alpha_3 K_1 K_2 K_3 K_7 = 38.4$	$K_7 = 2,$	$\alpha_3 = .96$

since

$$D(s) = s(s^3 + 4.16s^2 + 10.6s + 38.4)$$

Two transfer functions, $Y_{\dot{\phi}\delta_a}$ and $Y_{\dot{r}\delta_r}$ were investigated, as being representative. The value of the potentiometer setting corresponding to the highest degree 's' term in the numerator was found as in the longitudinal case. See Appendix C for details. The remaining potentiometers were adjusted giving the plots and variations shown in Fig. 9.

RESULTS AND CONCLUSIONS

By determining potentiometer settings corresponding to initial slope and also the steady state value, where available, it was possible to obtain accuracy in matching the curves such that the zeroes were determined with errors of less than 5%.

In the case of q/δ_e , where two potentiometer settings were unknown, only 18 trials were necessary to obtain this accuracy.

See Fig. 5 and Fig. 9 to observe the variation in response curves for variation in numerator potentiometer settings. It is concluded that the best general procedure is as follows:

1) Obtain estimates of poles and zeroes from pre-flight data.

2) Obtain poles from flight records involving step inputs and coefficients of first and last terms in numerator corresponding to initial slope and steady state, if possible. Then compare with previous estimates.

$$\lim_{t \to 0} d^{m-n}X/dt^{m-n} = b_n \delta / a_m$$

$$\lim_{t \to \infty} X(t) = b_o \delta / a_o$$

3) Adjust potentiometers to correspond to the known parameters.

4) Set estimates in those potentiometers which must be adjusted.

5) Make preliminary adjustments to observe which potentiometer has more effect in the case where two potentiometer settings are unknown.

6) Adjust the more critical to give loose matching, observing trend of frequency, phase shift, peak, and steady state values.

7) Adjust the remaining potentiometer for a closer match while observing trends.

8) Return to the first potentiometer and repeat adjustment.

9) Continue iterating for as close a match as desired.

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APPENDIX A

APPENDIX A

AIRCRAFT DATA AND STABILITY DERIVATIVES

The following data is given in references 4 and 5 for the F-86D aircraft at 30,000 feet, Mach 0.8, weight 15,000 lbs.

$S = 288\ ft^2$ $\quad c = 8.09\ ft$ $\quad b = 37.12\ ft$

$I_{xx} = 8625\ slug\ ft^2$ $\quad I_{yy} = 29400\ slug\ ft^2$

$I_{zz} = 35{,}300\ slug\ ft^2$ $\quad I_{zx} = 2{,}700\ slug\ ft^2$

$V_p = 796\ ft/sec$ $\quad C_{L_Q} = -.82/radian$

$\alpha = 2.3^{\circ} = 0.04\ rad$ $\quad C_{Y_P} =$ ———

$\rho = .00089\ slug/ft^3$ $\quad C_{Y_R} =$ ———

$C_L = 0.185$ $\quad C_{Y\delta_a} =$ ———

$C_{M_Q} = -6.5$ $\quad C_{Y_{\delta_r}} =$ ———

$C_{L_\alpha} = 4.62/radian$ $\quad C_{L_\beta} = -0.10/radian$

$C_{D_{C_L}} = .031$ $\quad C_{N_\beta} = .117$

$C_{M_\alpha} = -0.424$ $\quad C_{L_P} = -.433$

$C_{M_{\dot\alpha}} = -1.5$ $\quad C_{L_R} = .15$

$C_{D_\alpha} = .143$ $\quad C_{N_P} = -.01$

$C_D = .018$ $\quad C_{N_R} = -.2$

$C_{M_{\delta_e}} = -.0185/degree$ $\quad C_{L_{\delta_a}} = -.0029/degree$

$C_{L_{\delta_e}} = 0.0089/degree$ $\quad C_{L_{\delta_r}} = .00043/degree$

$C_{N_{\delta_a}} = -.000055/degree$

$C_{N_{\delta_r}} = -.001$

$C_{N_\beta} = .117$

$C_{L_\beta} = -.10$

APPENDIX B

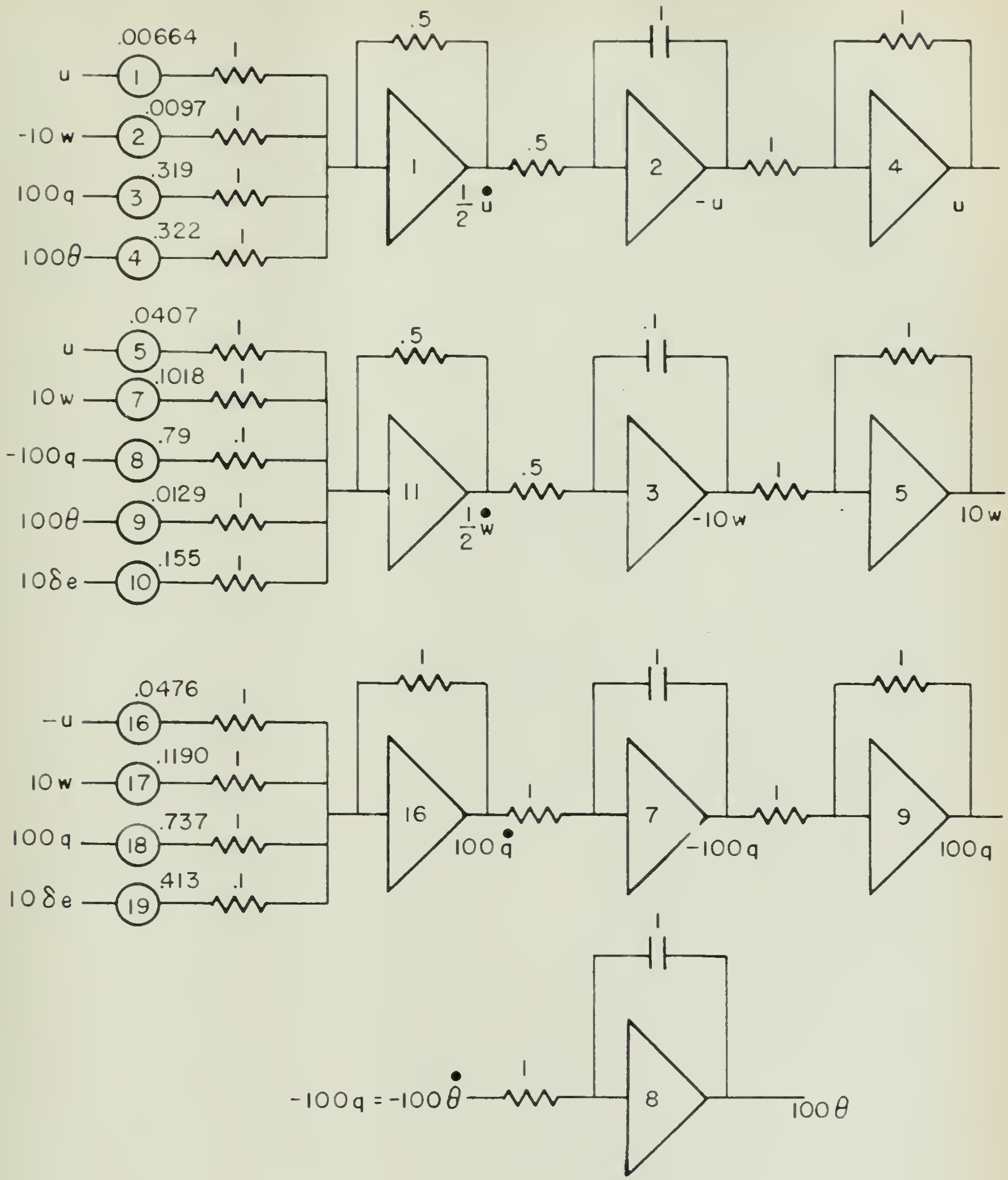


Fig. 2

Cumputer Circuit for Solving the Longitudinal Flight Equations

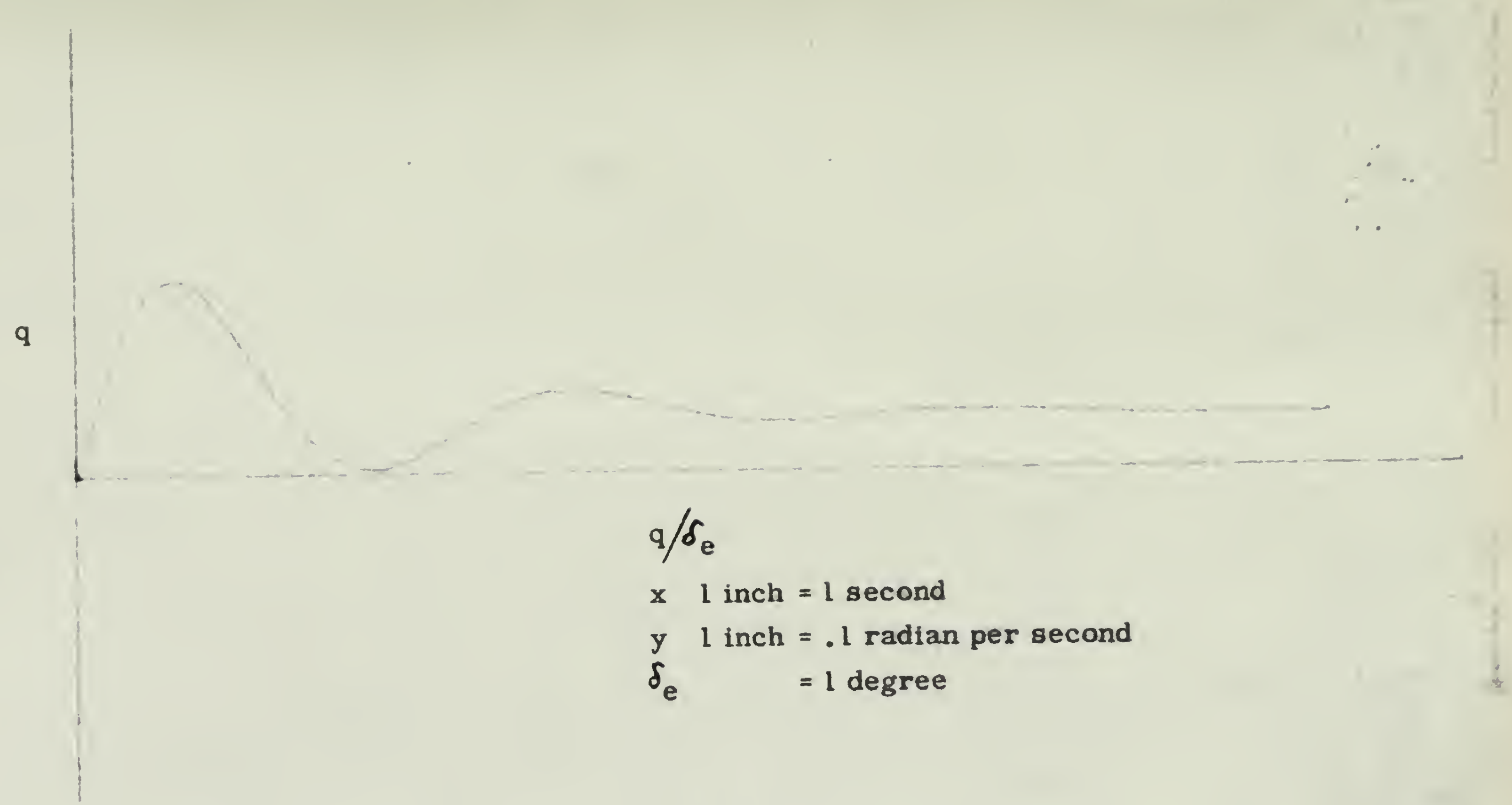
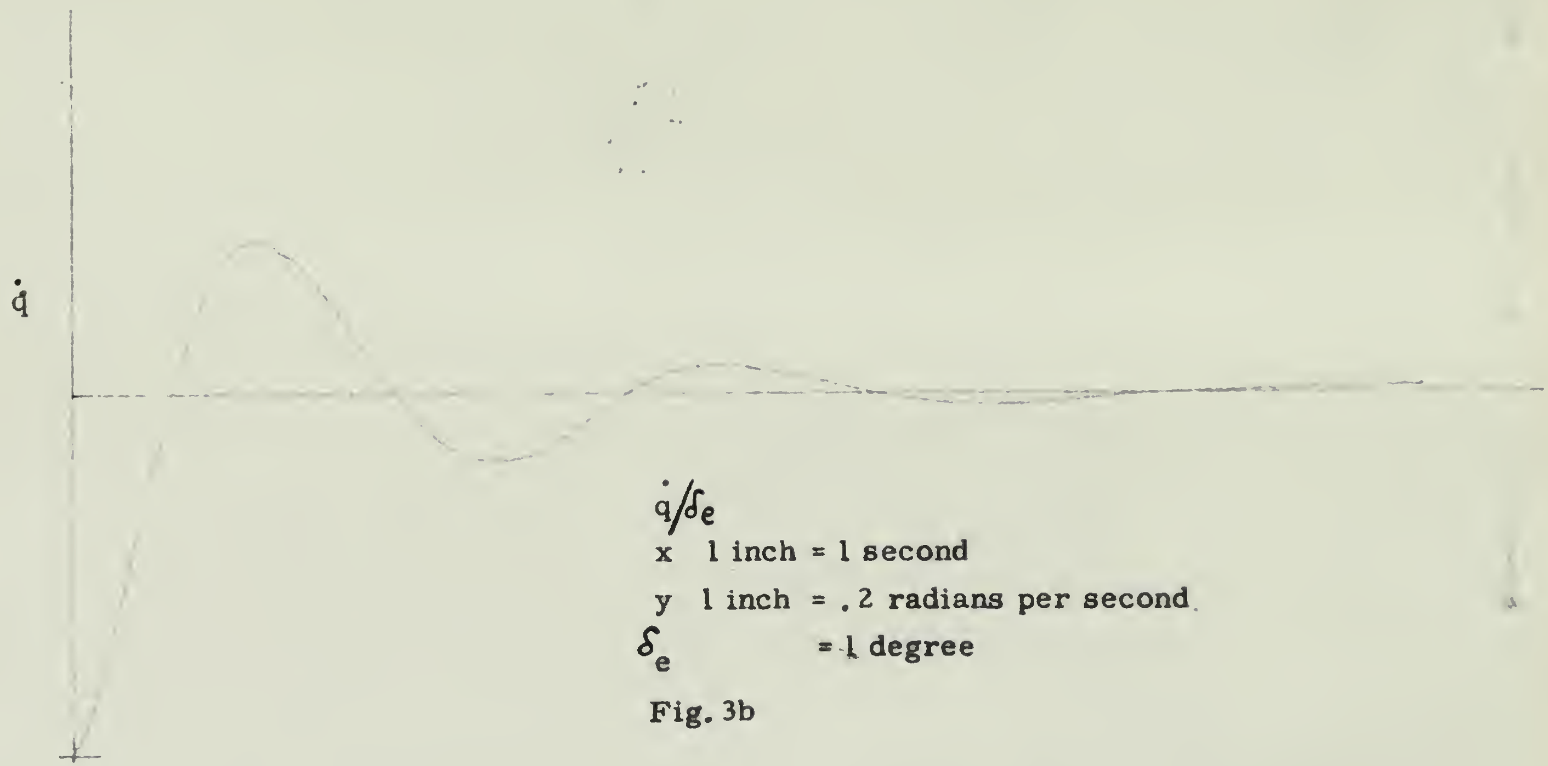

q
q/δ_e
x 1 inch = 1 second
y 1 inch = .1 radian per second
δ_e = 1 degree

Fig. 3a



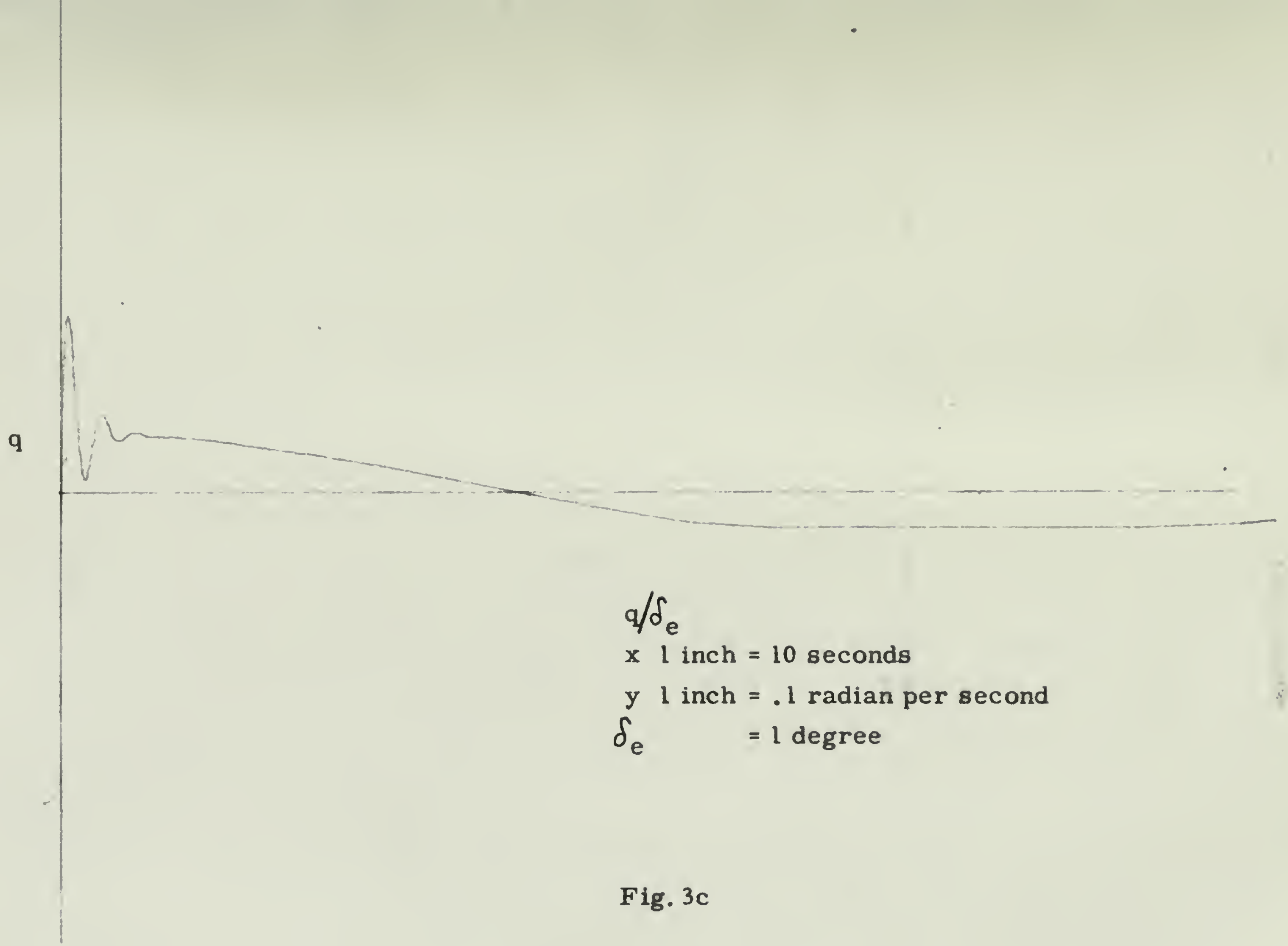
$\dot{q}/\delta_e$

x 1 inch = 1 second

y 1 inch = .2 radians per second

δ_e = 1 degree

Fig. 3b



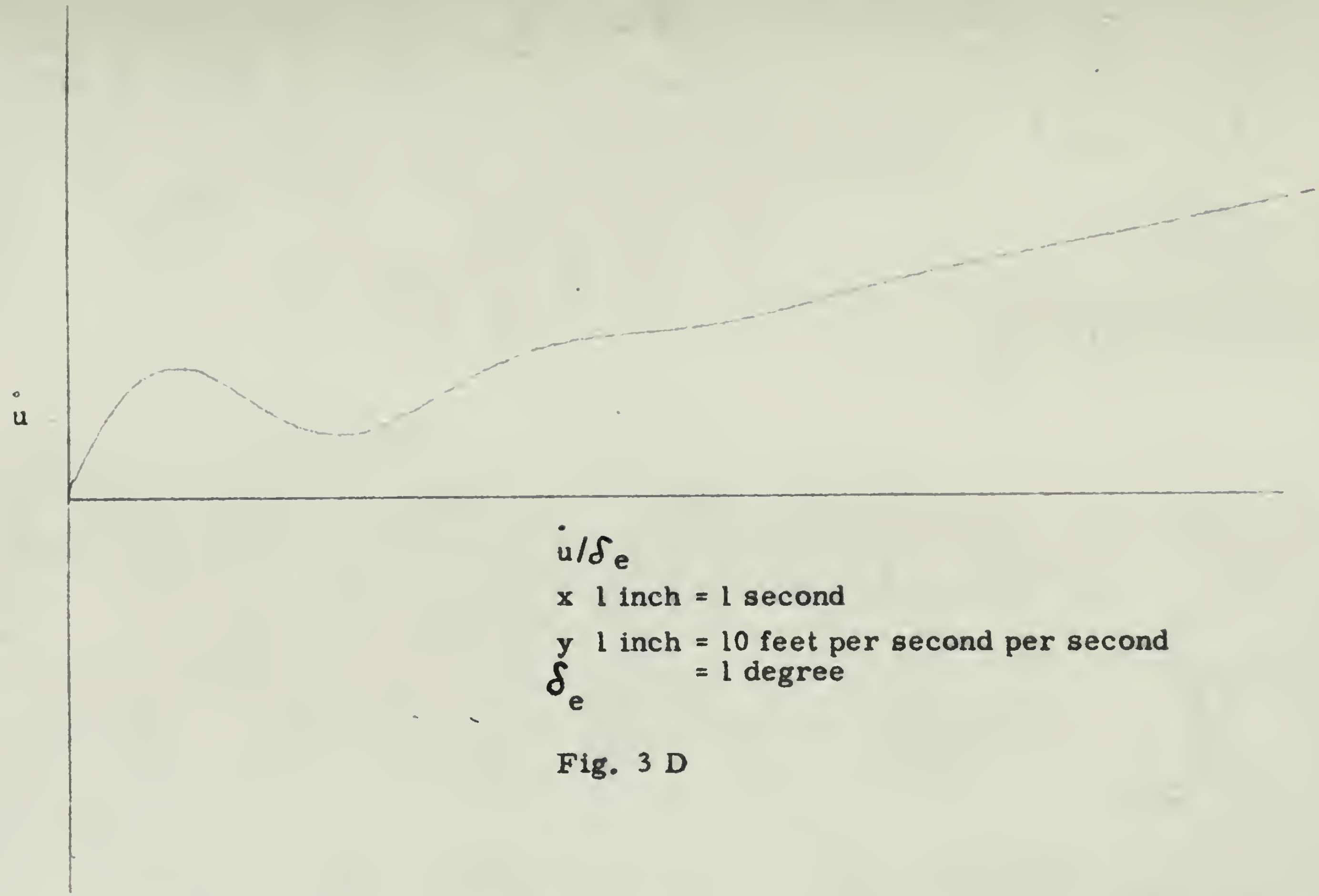
q/δ_e

x 1 inch = 10 seconds

y 1 inch = .1 radian per second

δ_e = 1 degree

Fig. 3c



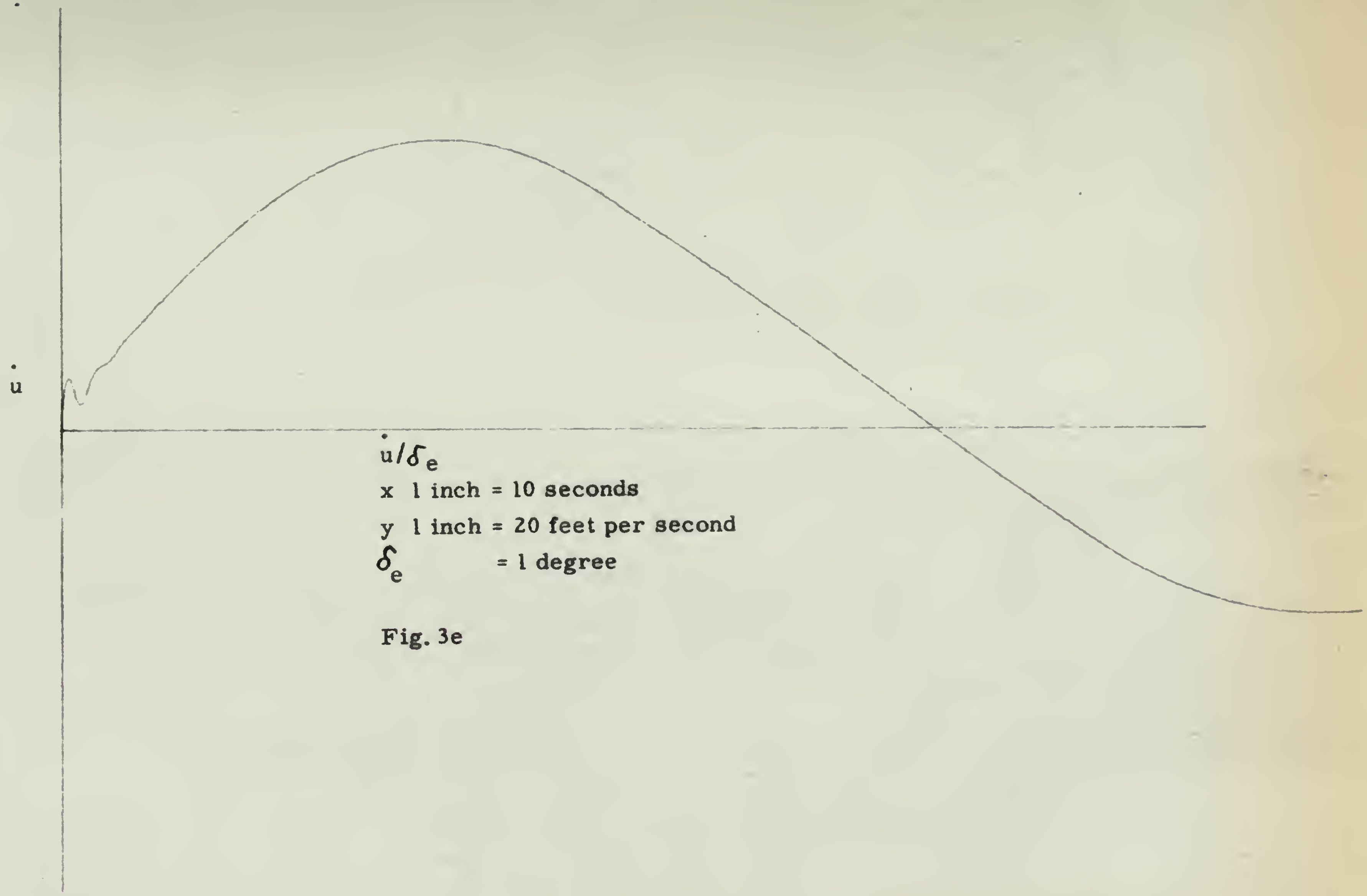
$\dot{u}/\delta_e$

x 1 inch = 1 second

y 1 inch = 10 feet per second per second

δ_e = 1 degree

Fig. 3 D



$\dot{u}/\delta_e$

x 1 inch = 10 seconds

y 1 inch = 20 feet per second

δ_e = 1 degree

Fig. 3e

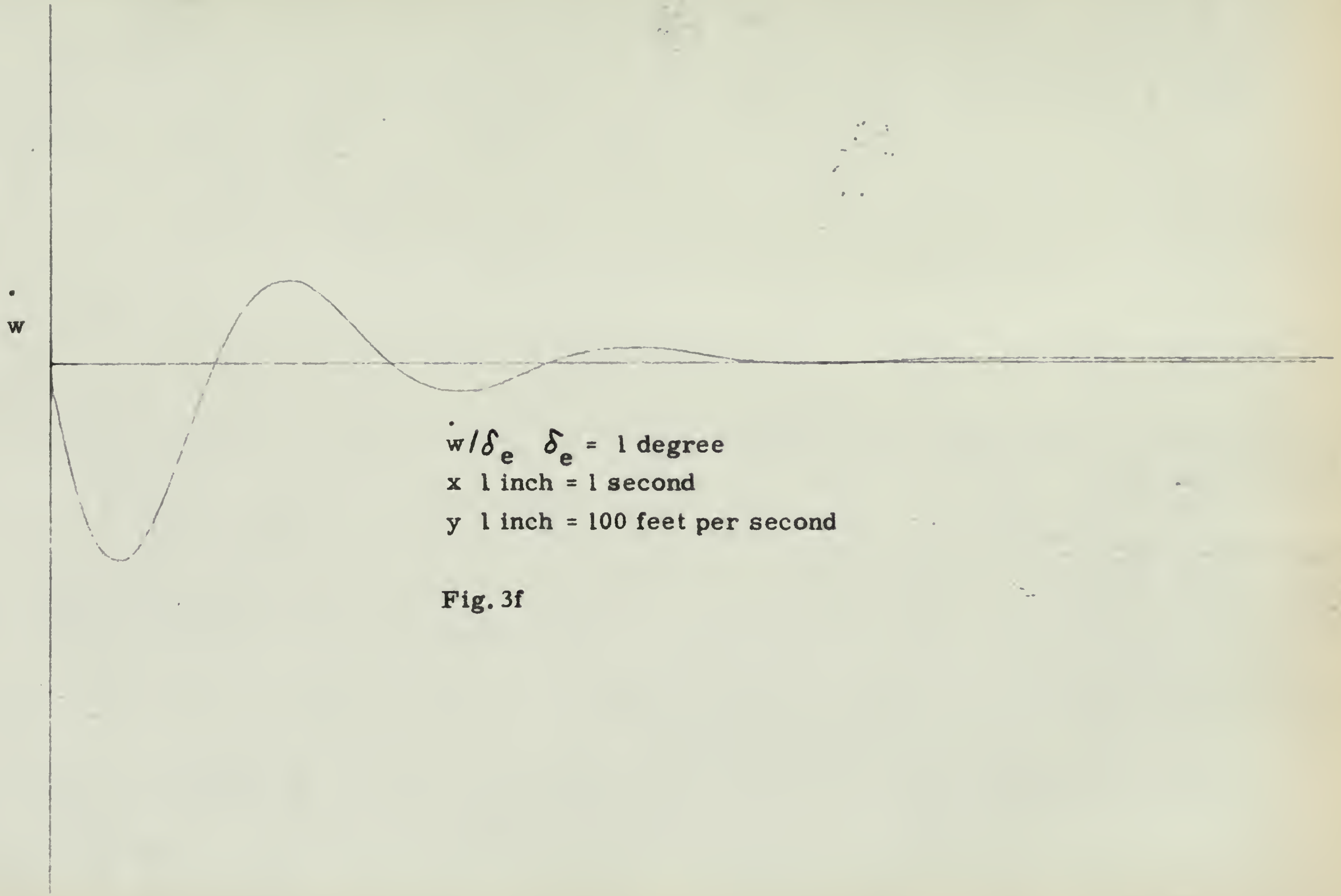

$\dot{w}$
$\dot{w}/\delta_e$ δ_e = 1 degree
x 1 inch = 1 second
y 1 inch = 100 feet per second

Fig. 3f

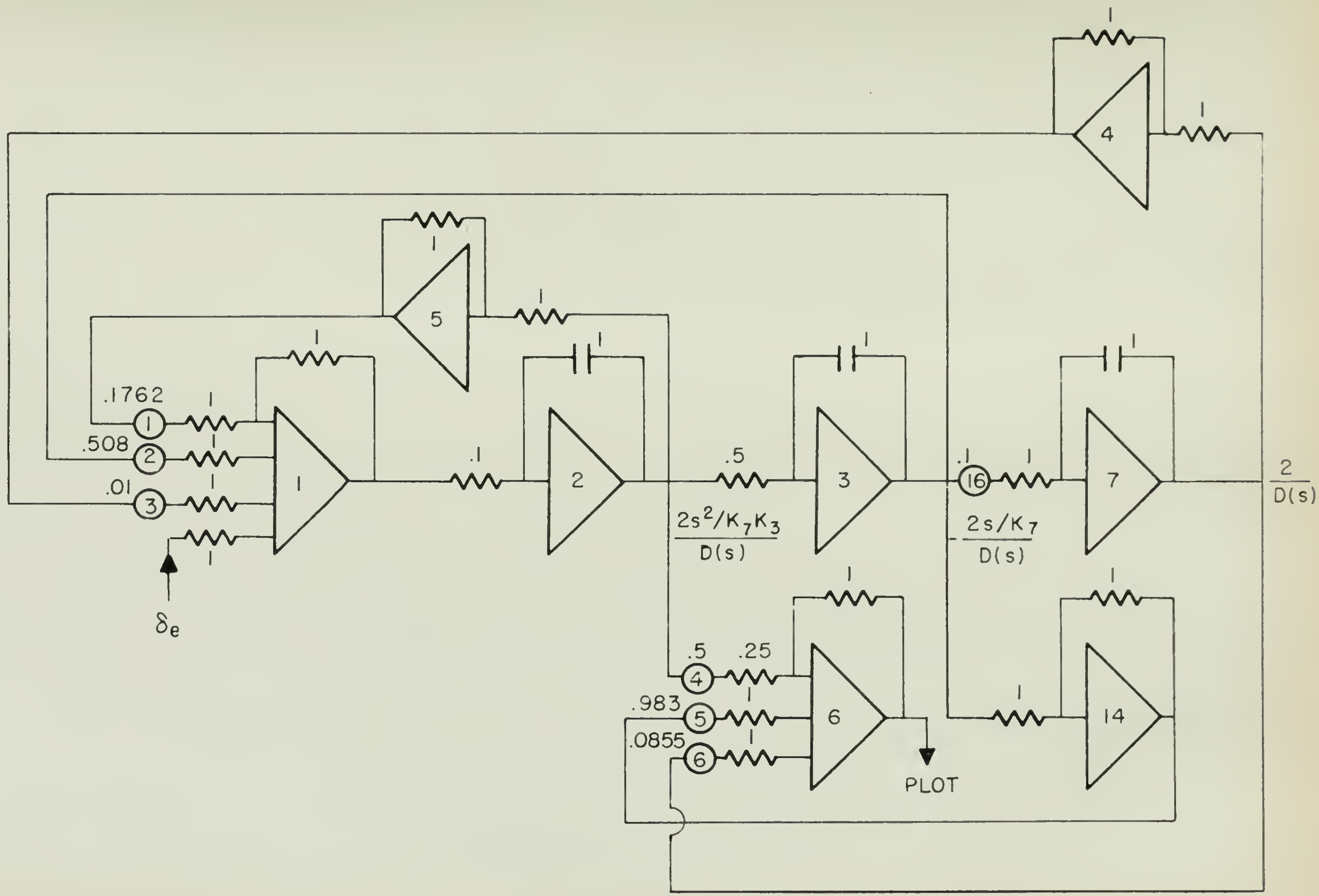
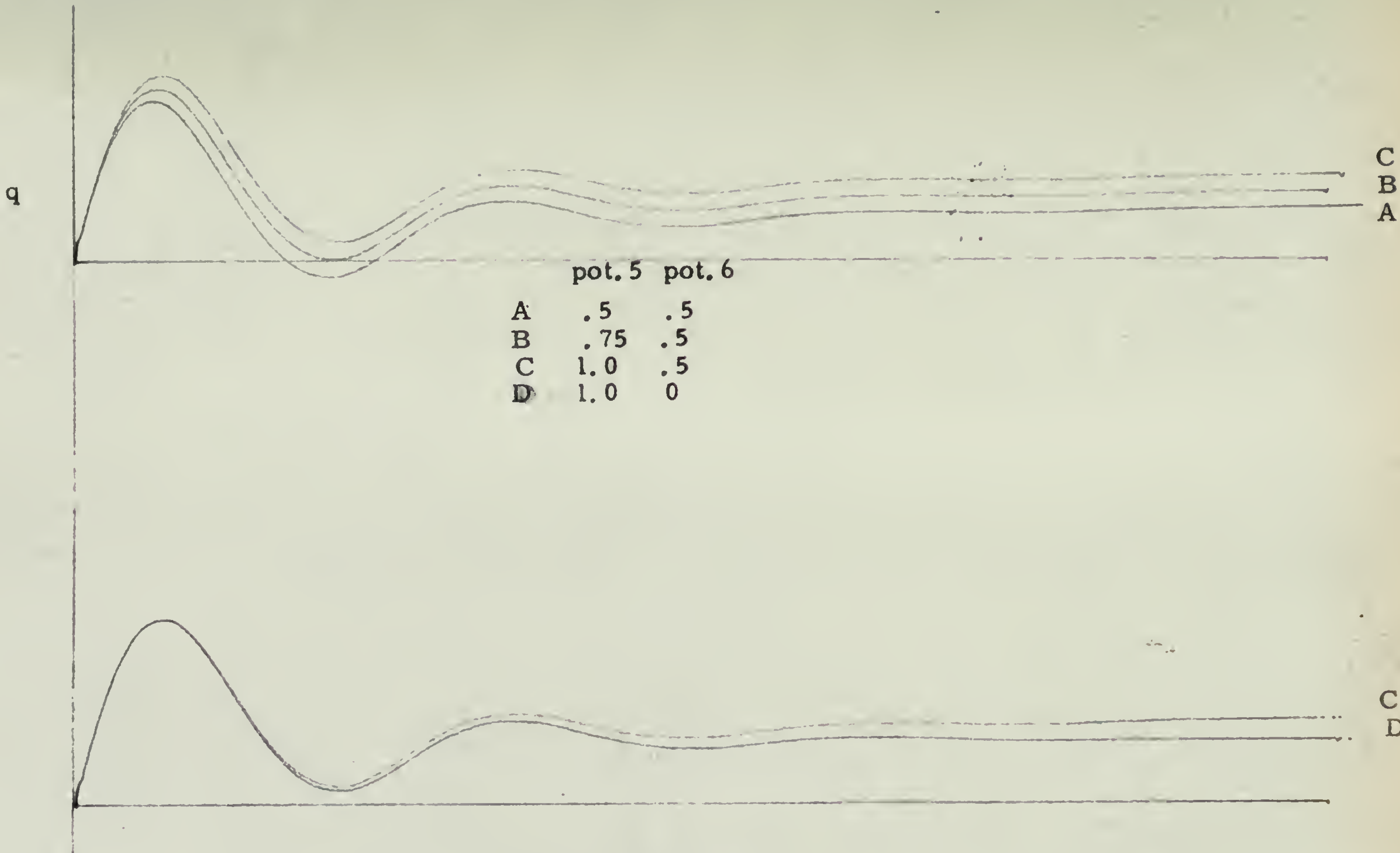


Fig. 4
Longitudinal Simulation for q/δ_e



q/δ_e Simulation

Fig. 5

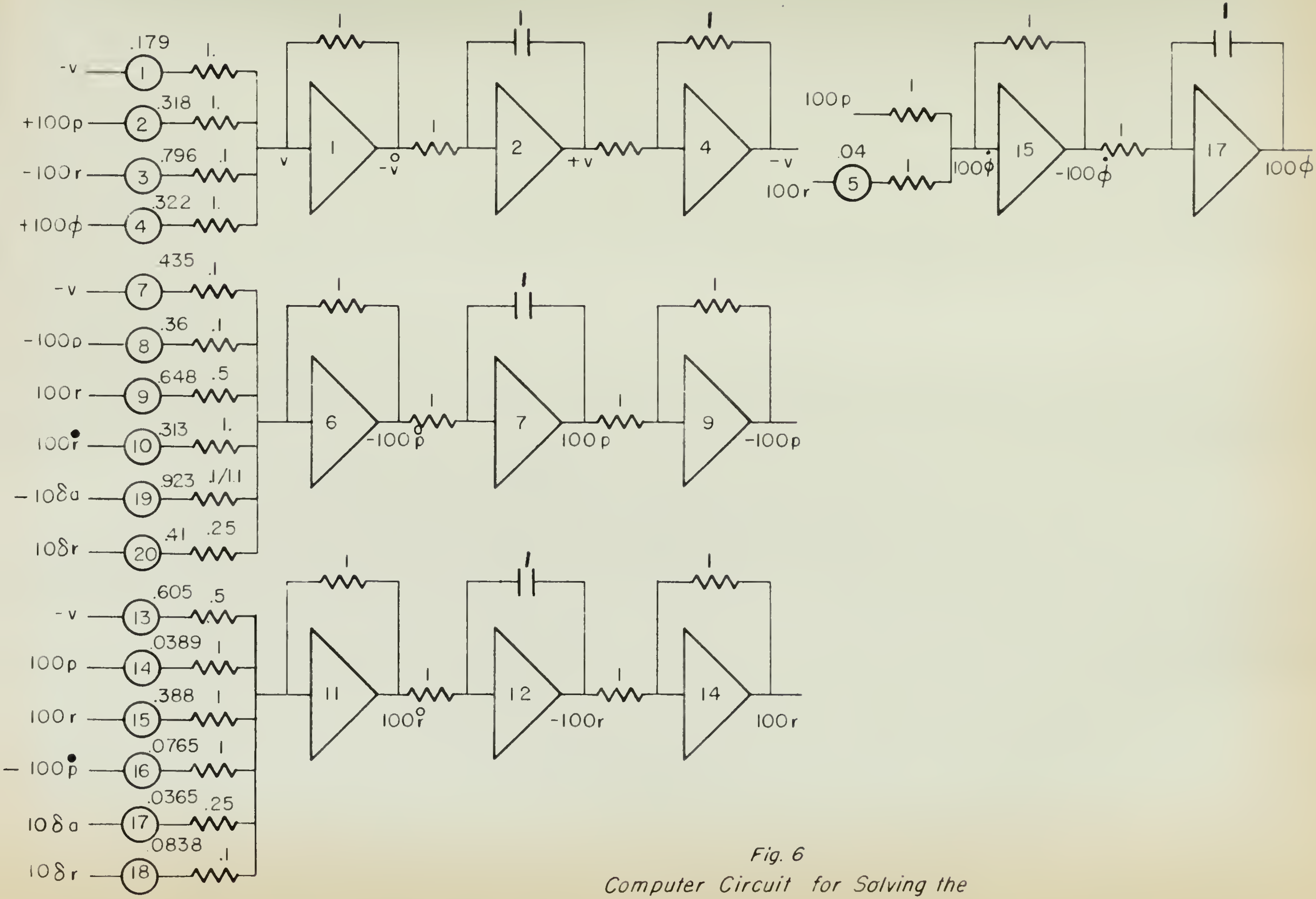
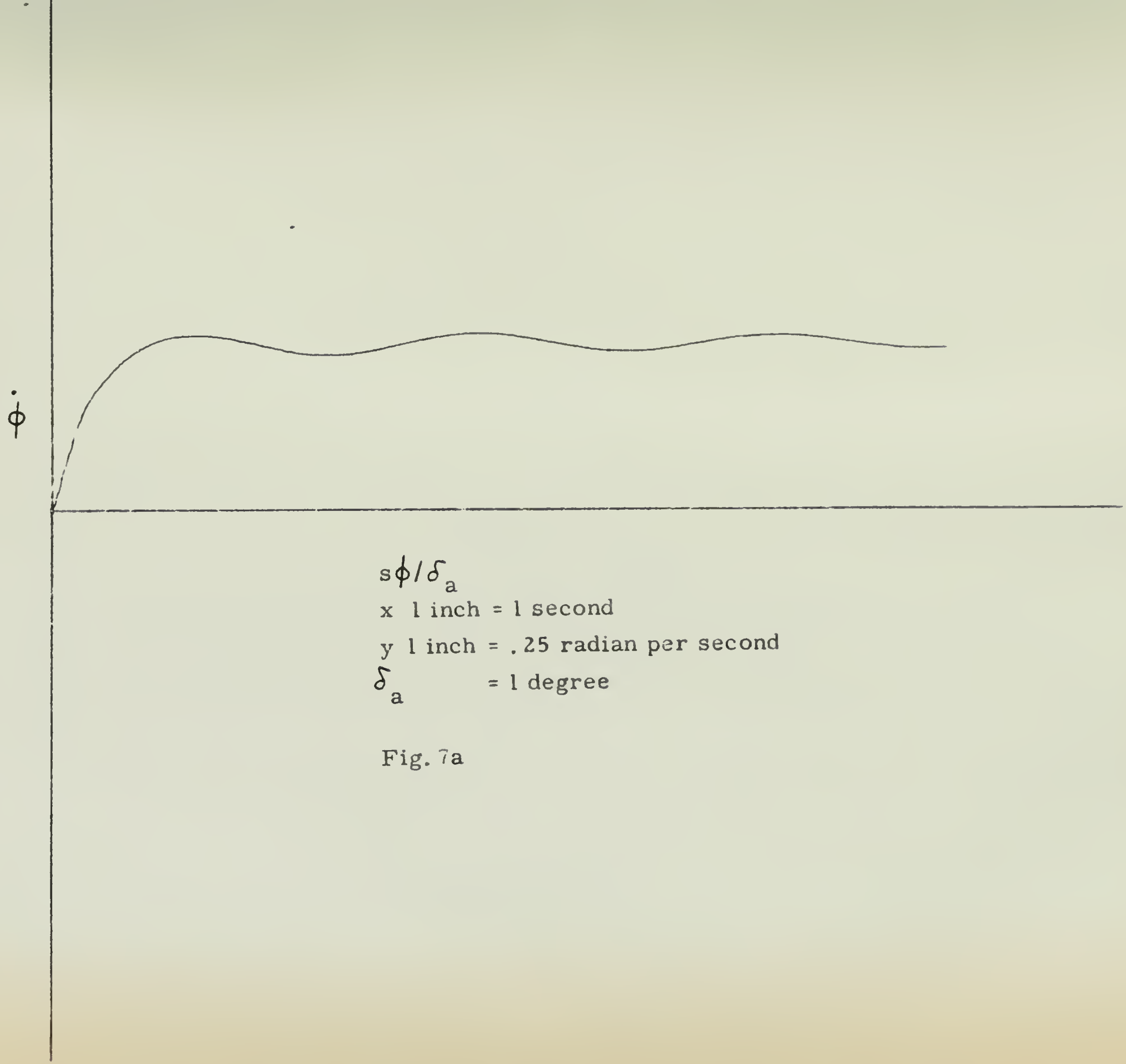


Fig. 6

Computer Circuit for Solving the Lateral Flight Equations



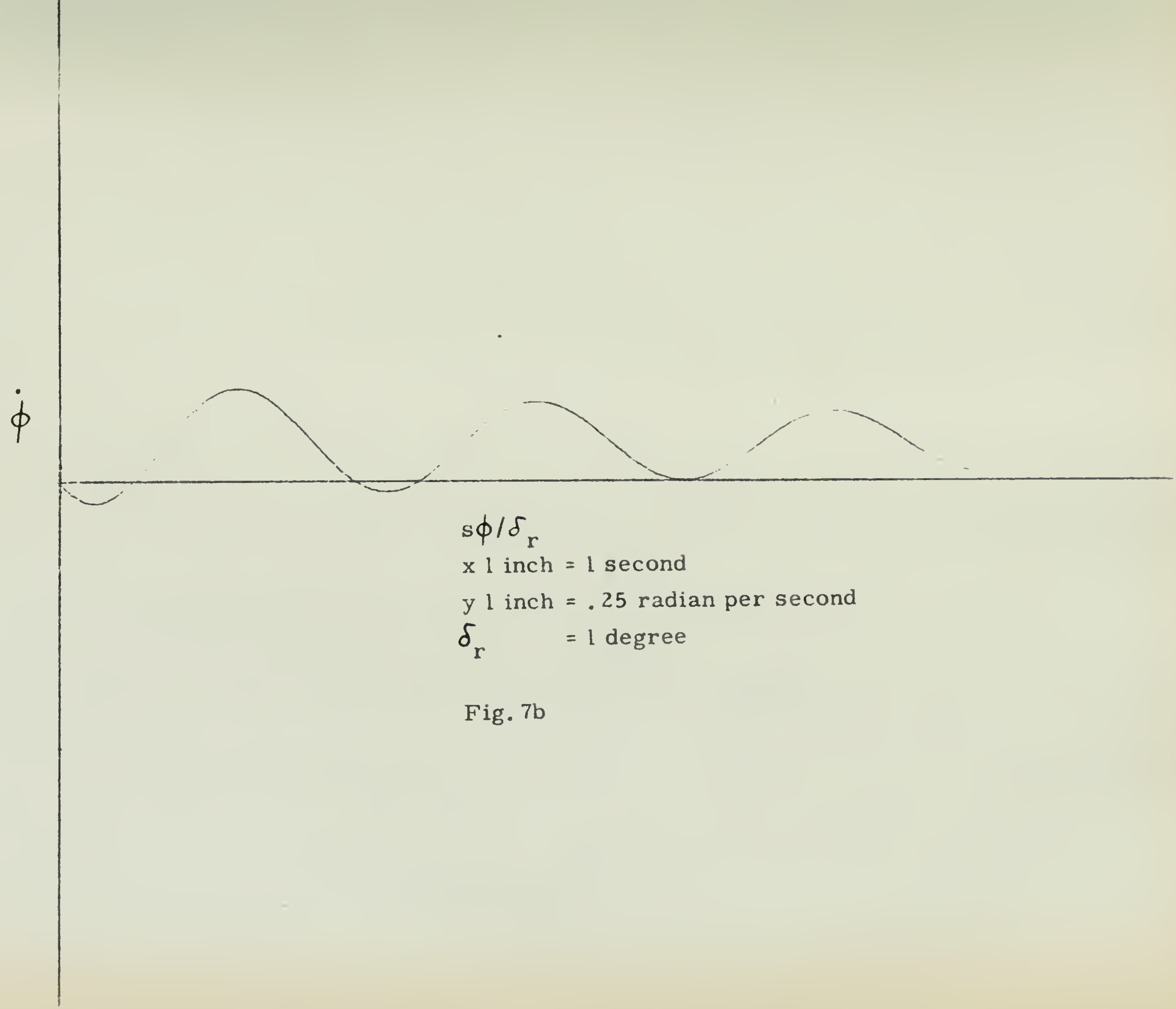
$s\phi/\delta_a$

x 1 inch = 1 second

y 1 inch = .25 radian per second

δ_a = 1 degree

Fig. 7a



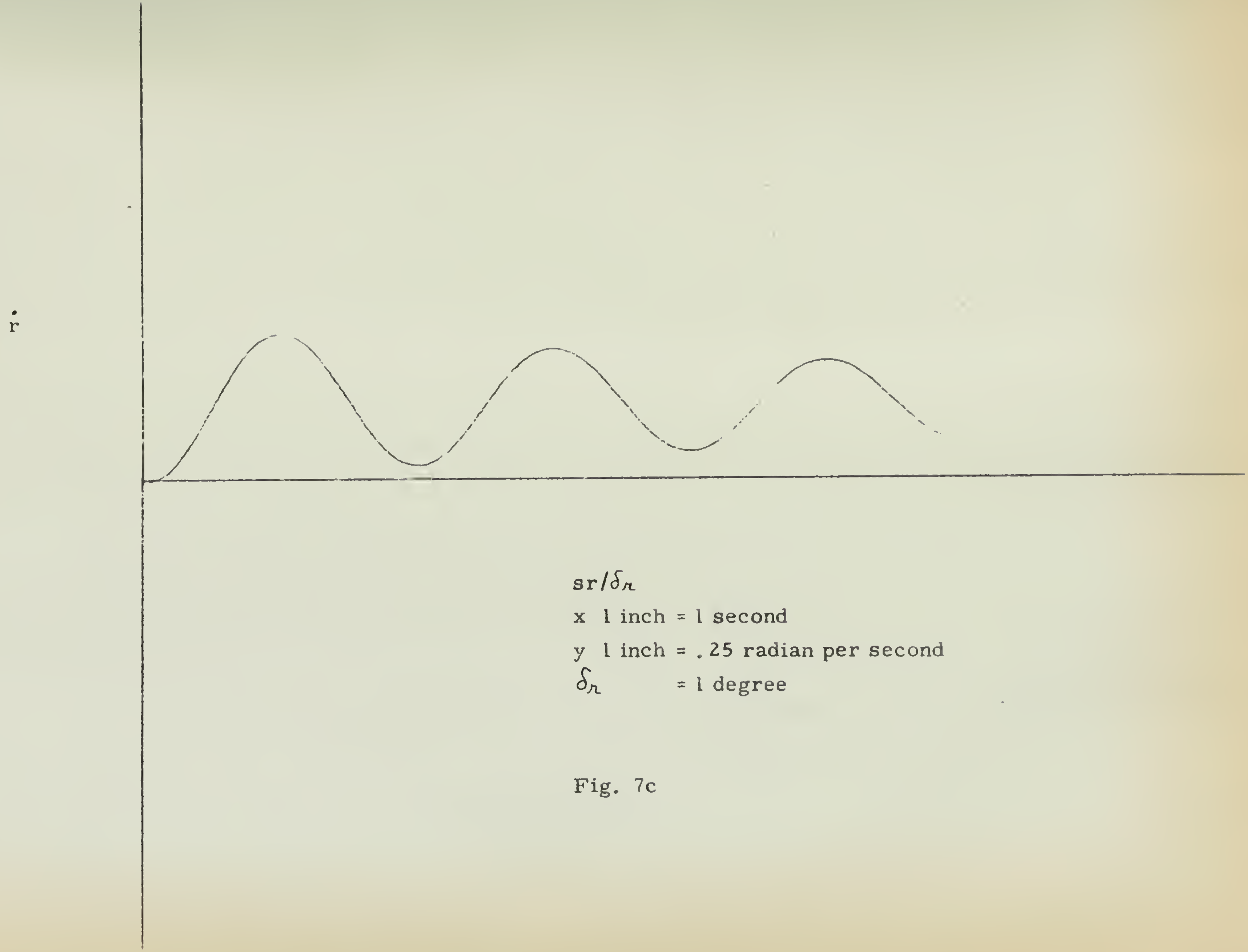
$s\phi/\delta_r$

x 1 inch = 1 second

y 1 inch = .25 radian per second

δ_r = 1 degree

Fig. 7b



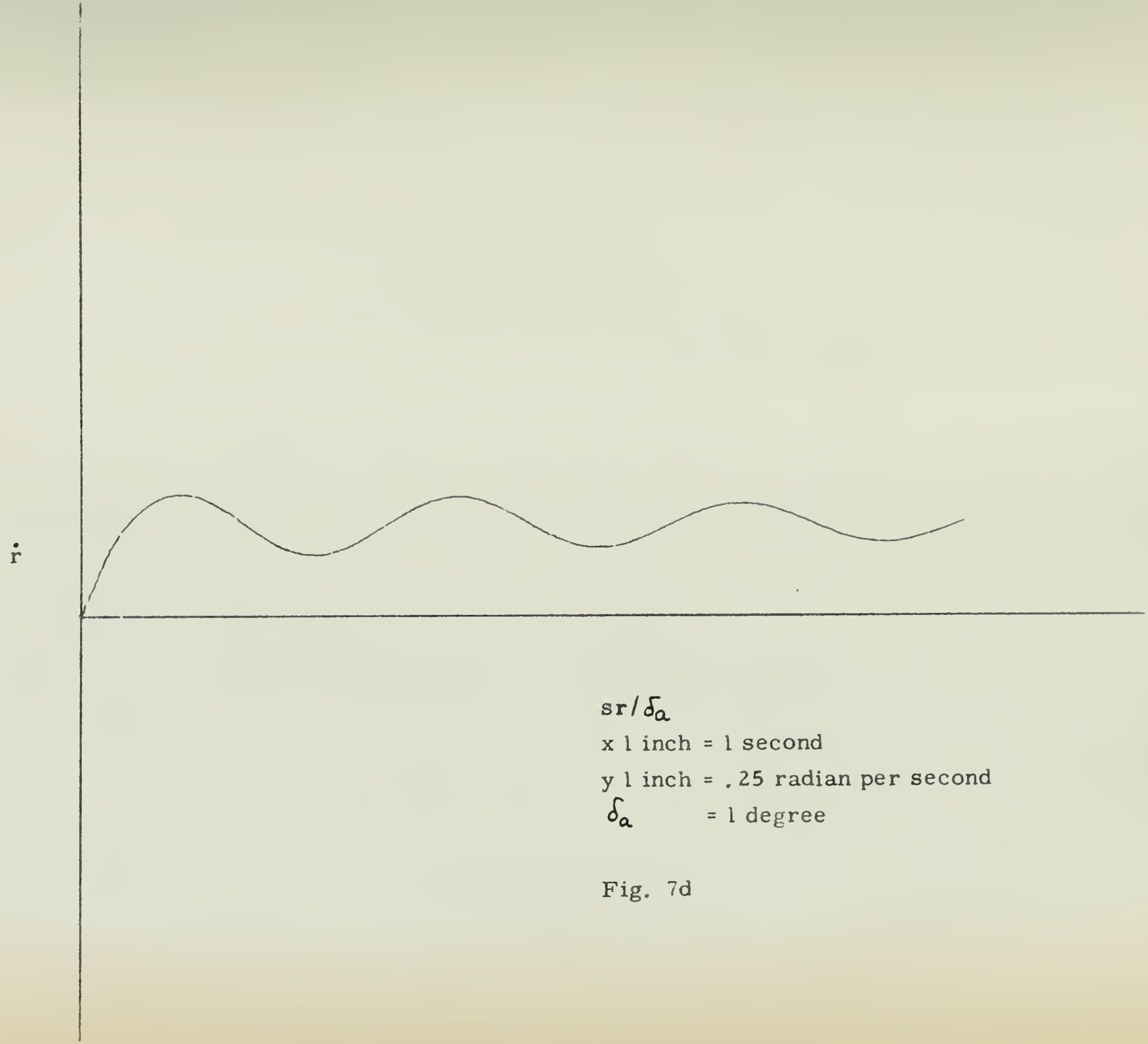
sr/δ_r

x 1 inch = 1 second

y 1 inch = .25 radian per second

δ_r = 1 degree

Fig. 7c



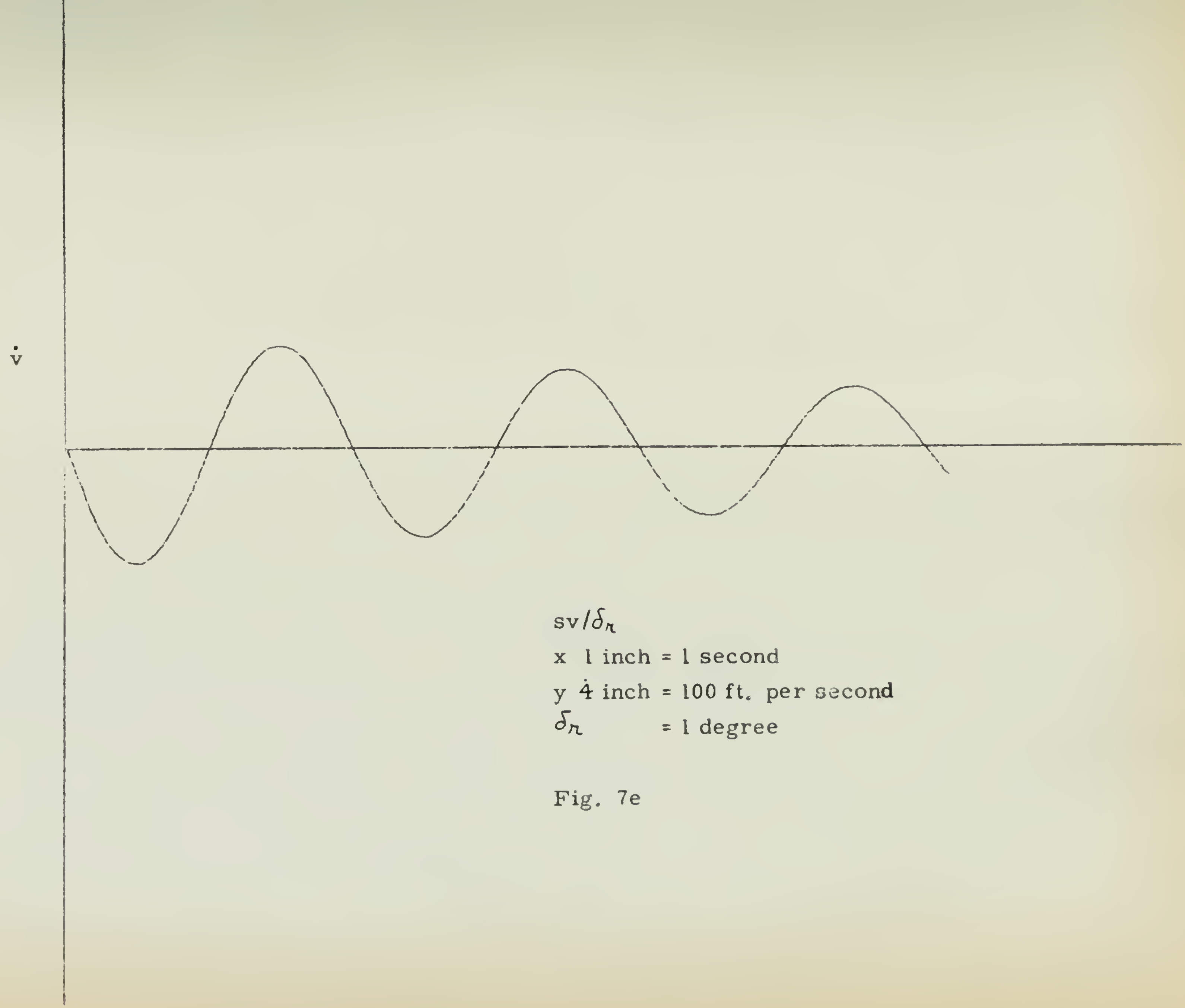
sr/δ_a

x 1 inch = 1 second

y 1 inch = .25 radian per second

δ_a = 1 degree

Fig. 7d



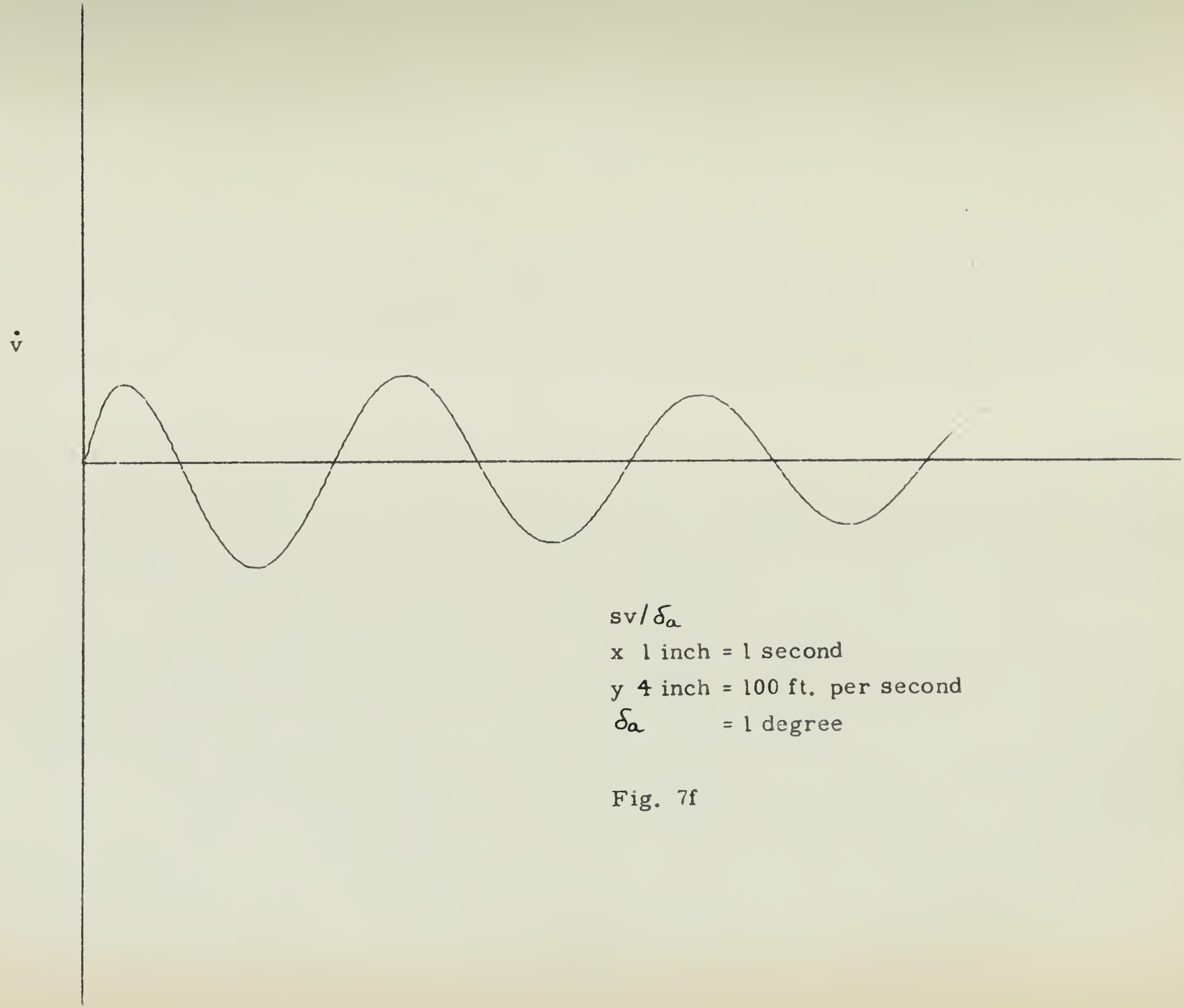
sv/δ_r

x 1 inch = 1 second

y $\dot{4}$ inch = 100 ft. per second

δ_r = 1 degree

Fig. 7e



sv/δ_a

x 1 inch = 1 second

y 4 inch = 100 ft. per second

δ_a = 1 degree

Fig. 7f

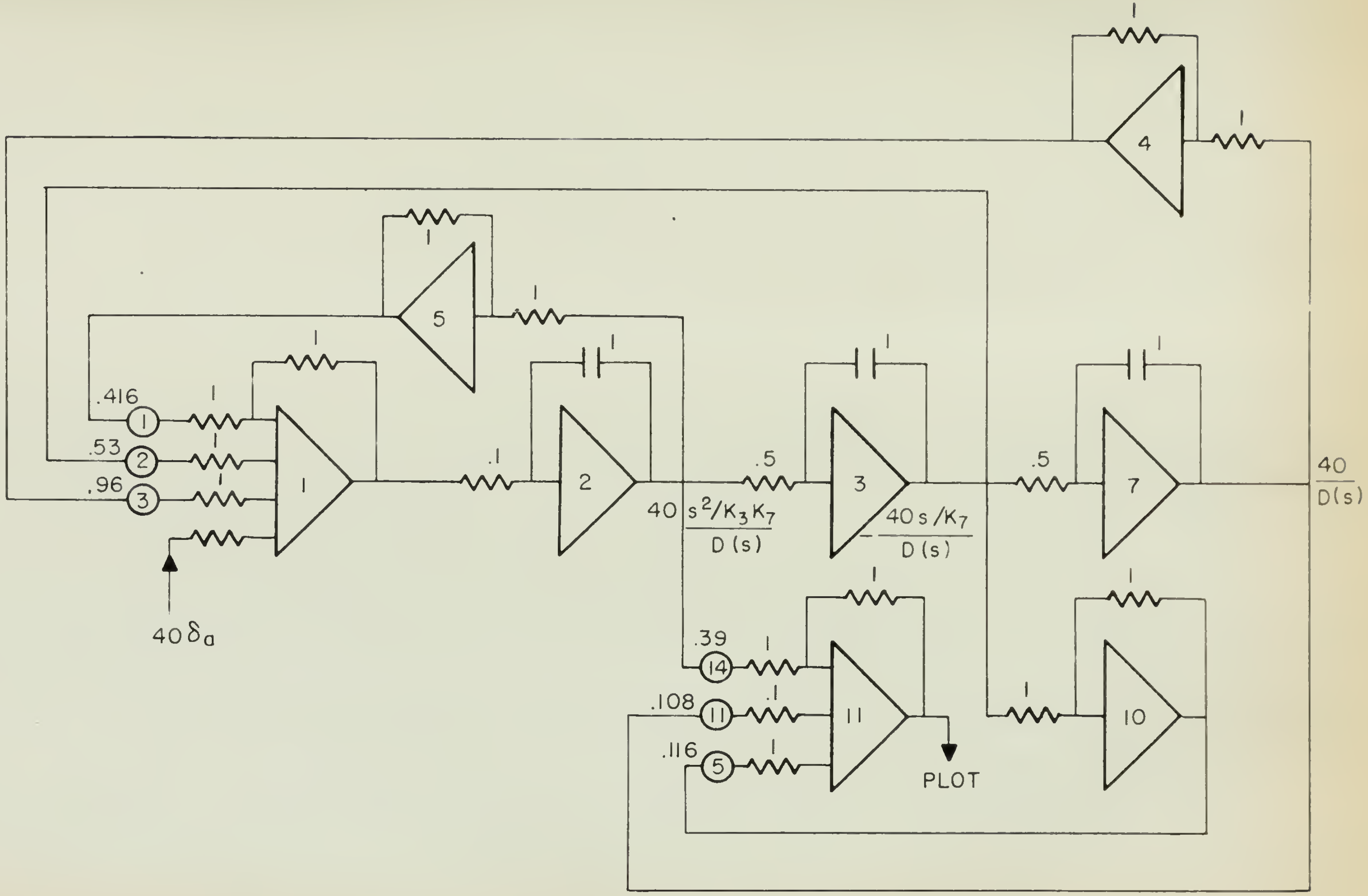


Fig. 8a
Lateral Simulation for $s\phi/\delta_a$

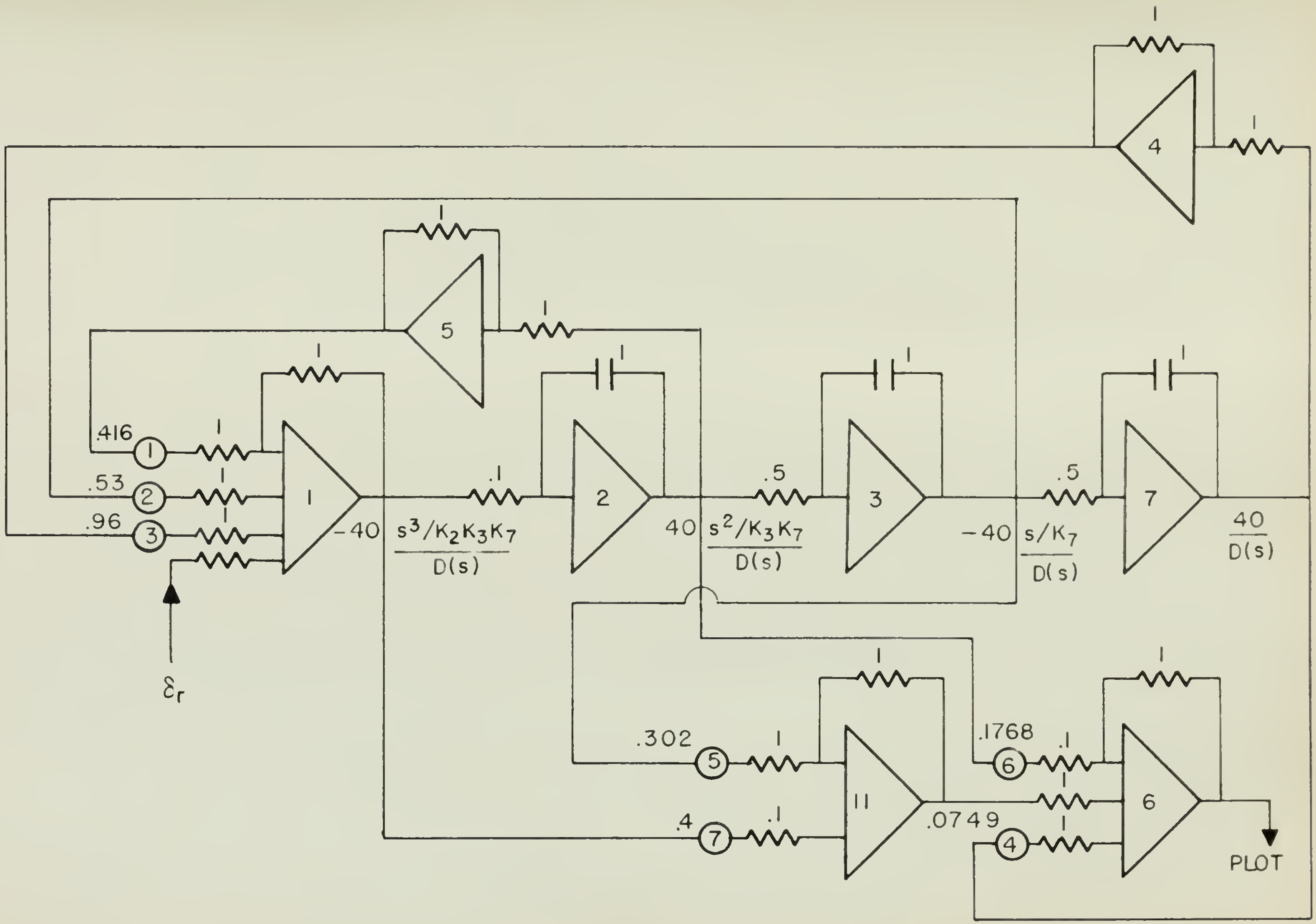
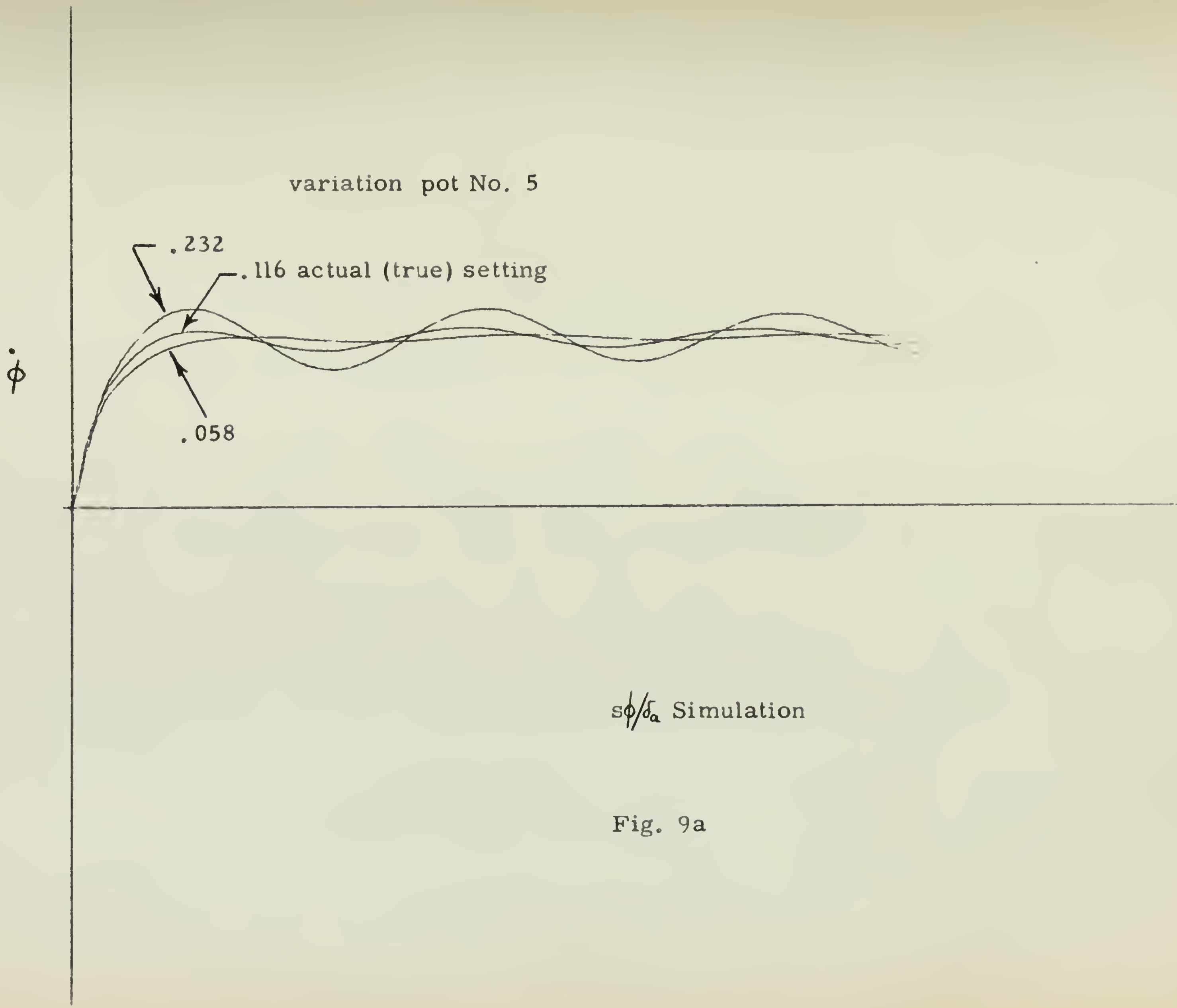
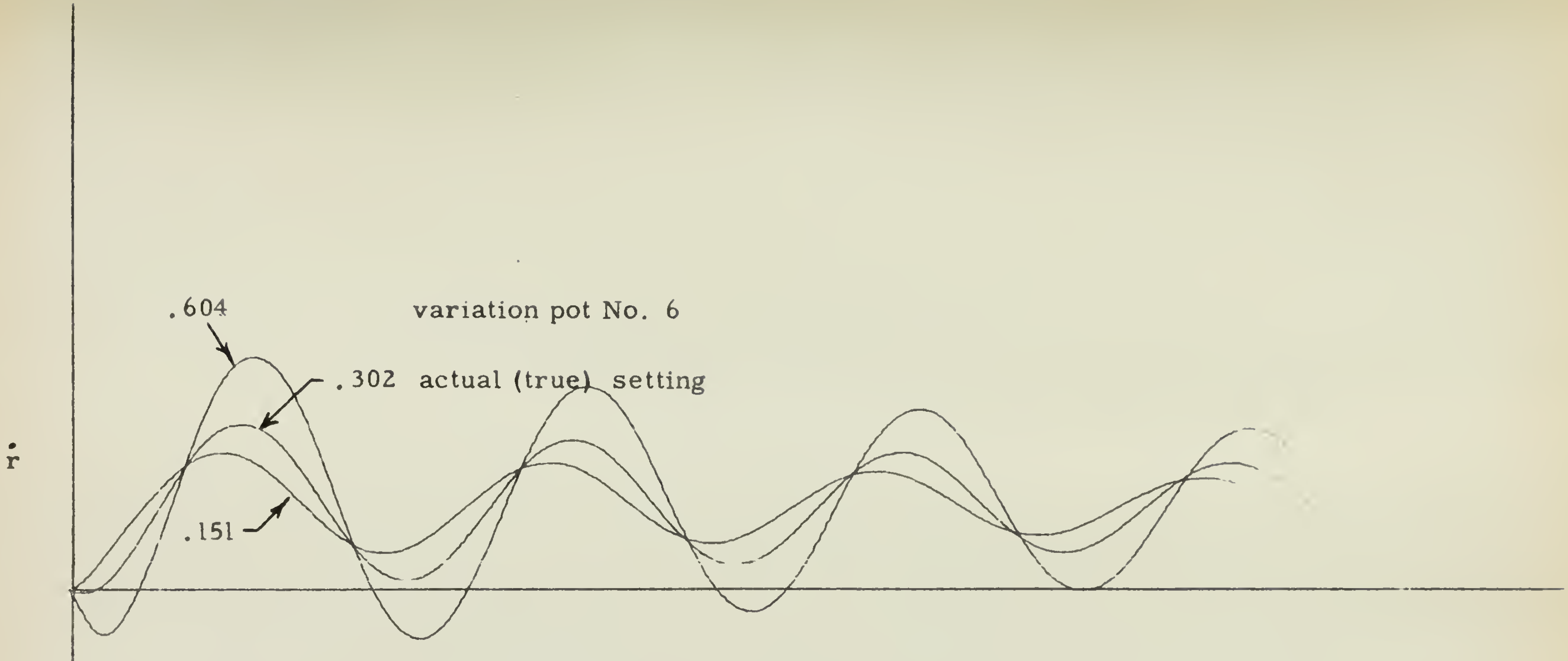


Fig. 8b
Lateral Simulation for sr/δ_r



$s\phi/\delta_a$ Simulation

Fig. 9a



$sr/\delta r$ Simulation

Fig. 9b

APPENDIX C

APPENDIX C

Determination of Potentiometer Settings

For the longitudinal case we found that

$$b_2 = .4 \text{ rad/sec}^2/\text{degree.}$$

from Fig. 3 a or 3b.

The scale that we wish the plotter to have is

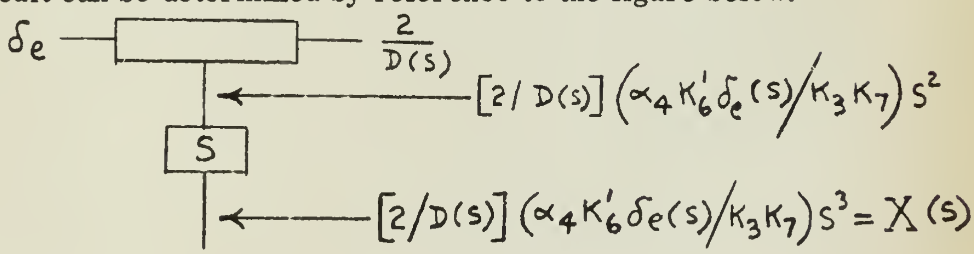
$$1 \text{ inch} = .1 \text{ rad/sec}^2$$

with the pen scale factor set to 10 volts per inch.

For a one degree step input the initial $\dot{q}$ should be equal to 40 volts, since

$$40 \text{ volts} = .4 \text{ rad/sec}^2.$$

The equation relating the 40 volts to the parameters of the circuit can be determined by reference to the figure below.



$$\lim_{s \to \infty} sX(s) = \dot{q}_o = (2/a_3)\,(\alpha_4 K_6' \delta_e/K_3K_7) = 40 \text{ volts}$$

For $\delta_e = 2$ volts

$$\alpha_4 K_6' = 2$$

In brief then for the initial slope

$$(K/a_3)\,(\alpha_4 K_6' \delta_e/K_3K_7) = (K_{vi}/K_r i)\,(b_2/a_3)$$

and for steady state

$$(K/a_o)\,(\alpha_6 K_6' \delta_e) = (K_{vi}/K_{ri})\,(b_o/a_o)$$

where

K = denominator multiplying factor

K_{vi} = the pen scale factor on the plotter (in the above example it was ten volts per inch.)

K_{ri} = the number of radians /second or the number of radians/second /second (in the above example it was .1)